SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q. 10 carry one mark each.

Q. 1 Let G be a finite group. Then G is necessarily a cyclic group if the order of G is

(A) 4

(B) 7

(C) 6

(D) 10

Q. 2 Let $\mathbf{v}_1, \ldots, \mathbf{v}_9$ be the column vectors of a non-zero 9×9 real matrix A. Let $a_1, \ldots, a_9 \in \mathbb{R}$, not all zero, be such that $\sum_{i=1}^{9} a_i \mathbf{v}_i = \mathbf{0}$. Then the system $A\mathbf{x} = \sum_{i=1}^{9} \mathbf{v}_i$ has

(A) no solution

(B) a unique solution

(C) more than one but only finitely many solutions

(D) infinitely many solutions

Q. 3 Which of the following is a subspace of the real vector space $\mathbb{R}^3$?

(A) $\{(x, y, z) \in \mathbb{R}^3 : (y + z)^2 + (2x - 3y)^2 = 0\}$

(B) $\{(x, y, z) \in \mathbb{R}^3 : y \in \mathbb{Q}\}$

(C) $\{(x, y, z) \in \mathbb{R}^3 : yz = 0\}$

(D) $\{(x, y, z) \in \mathbb{R}^3 : x + 2y - 3z + 1 = 0\}$

Q. 4 Consider the initial value problem

$$\frac{dy}{dx} + \alpha y = 0,$$
$$y(0) = 1,$$

where $\alpha \in \mathbb{R}$. Then

(A) there is an α such that $y(1) = 0$

(B) there is a unique α such that $\lim_{x\to\infty} y(x) = 0$

(C) there is NO α such that $y(2) = 1$

(D) there is a unique α such that $y(1) = 2$

Q. 5 Let $p(x) = x^{57} + 3x^{10} - 21x^3 + x^2 + 21$ and

$$q(x) = p(x) + \sum_{j=1}^{57} p^{(j)}(x) \quad \text{for all } x \in \mathbb{R},$$

where $p^{(j)}(x)$ denotes the j^{th} derivative of $p(x)$. Then the function q admits

(A) NEITHER a global maximum NOR a global minimum on $\mathbb{R}$

(B) a global maximum but NOT a global minimum on $\mathbb{R}$

(C) a global minimum but NOT a global maximum on $\mathbb{R}$

(D) a global minimum and a global maximum on $\mathbb{R}$

Q. 6 The limit

$$\lim_{a \to 0} \left(\frac{\int_0^a \sin(x^2)\, dx}{\int_0^a (\ln(x+1))^2\, dx} \right)$$

is

(A) 0

(B) 1

(C) $\frac{\pi}{e}$

(D) non-existent

Q. 7 The value of

$$\int_0^1 \int_0^{1-x} \cos(x^3+y^2)\,dy\,dx - \int_0^1 \int_0^{1-y} \cos(x^3+y^2)\,dx\,dy$$

is

(A) 0

(B) $\frac{\cos(1)}{2}$

(C) $\frac{\sin(1)}{2}$

(D) $\cos\left(\frac{1}{2}\right) - \sin\left(\frac{1}{2}\right)$

Q. 8 Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x,y) = (e^x \cos(y), e^x \sin(y))$. Then the number of points in $\mathbb{R}^2$ that do NOT lie in the range of f is

(A) 0

(B) 1

(C) 2

(D) infinite

Q. 9 Let $a_n = \left(1 + \frac{1}{n}\right)^n$ and $b_n = n \cos\left(\frac{n!\pi}{2^{10}}\right)$ for $n \in \mathbb{N}$. Then

(A) (a_n) is convergent and (b_n) is bounded

(B) (a_n) is NOT convergent and (b_n) is bounded

(C) (a_n) is convergent and (b_n) is unbounded

(D) (a_n) is NOT convergent and (b_n) is unbounded

Q. 10 Let (a_n) be a sequence of real numbers defined by

$$a_n = \begin{cases} 1 & \text{if } n \text{ is prime} \\ -1 & \text{if } n \text{ is not prime.} \end{cases}$$

Let $b_n = \frac{a_n}{n}$ for $n \in \mathbb{N}$. Then

(A) both (a_n) and (b_n) are convergent

(B) (a_n) is convergent but (b_n) is NOT convergent

(C) (a_n) is NOT convergent but (b_n) is convergent

(D) both (a_n) and (b_n) are NOT convergent

Q. 11 – Q. 30 carry two marks each.

Q. 11 Let $a_n = \sin\left(\frac{1}{n^3}\right)$ and $b_n = \sin\left(\frac{1}{n}\right)$ for $n \in \mathbb{N}$. Then

(A) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent

(B) $\sum_{n=1}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} b_n$ is NOT convergent

(C) $\sum_{n=1}^{\infty} a_n$ is NOT convergent but $\sum_{n=1}^{\infty} b_n$ is convergent

(D) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are NOT convergent

Q. 12 Consider the following statements:

I. There exists a linear transformation from $\mathbb{R}^3$ to itself such that its range space and null space are the same.

II. There exists a linear transformation from $\mathbb{R}^2$ to itself such that its range space and null space are the same.

Then

(A) both I and II are TRUE

(B) I is TRUE but II is FALSE

(C) II is TRUE but I is FALSE

(D) both I and II are FALSE

Q. 13 Let

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -2 & 2 & 2 \end{pmatrix}$$

and $B = A^5 + A^4 + I_3$. Which of the following is NOT an eigenvalue of B?

(A) 1

(B) 2

(C) 49

(D) 3

Q. 14 The system of linear equations in x_1, x_2, x_3

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & \alpha \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ \beta \end{pmatrix}$$

where $\alpha, \beta \in \mathbb{R}$, has

(A) at least one solution for any α and β

(B) a unique solution for any β when $\alpha \neq 1$

(C) NO solution for any α when $\beta \neq 5$

(D) infinitely many solutions for any α when $\beta = 5$

Q. 15 Let S and T be non-empty subsets of $\mathbb{R}^2$, and W be a non-zero proper subspace of $\mathbb{R}^2$. Consider the following statements:

I. If $\operatorname{span}(S) = \mathbb{R}^2$, then $\operatorname{span}(S \cap W) = W$.

II. $\operatorname{span}(S \cup T) = \operatorname{span}(S) \cup \operatorname{span}(T)$.

Then

(A) both I and II are TRUE

(B) I is TRUE but II is FALSE

(C) II is TRUE but I is FALSE

(D) both I and II are FALSE

Q. 16 Let $f(x, y) = e^{x^2+y^2}$ for $(x, y) \in \mathbb{R}^2$, and a_n be the determinant of the matrix

$$\begin{pmatrix} \dfrac{\partial^2 f}{\partial x^2} & \dfrac{\partial^2 f}{\partial x \partial y} \\[2ex] \dfrac{\partial^2 f}{\partial y \partial x} & \dfrac{\partial^2 f}{\partial y^2} \end{pmatrix}$$

evaluated at the point $(\cos(n), \sin(n))$. Then the limit $\lim\limits_{n\to\infty} a_n$ is

(A) non-existent

(B) 0

(C) $6e^2$

(D) $12e^2$

Q. 17 Let $f(x,y) = \ln(1+x^2+y^2)$ for $(x,y) \in \mathbb{R}^2$. Define

$$P = \left.\frac{\partial^2 f}{\partial x^2}\right|_{(0,0)} \qquad Q = \left.\frac{\partial^2 f}{\partial x \partial y}\right|_{(0,0)}$$

$$R = \left.\frac{\partial^2 f}{\partial y \partial x}\right|_{(0,0)} \qquad S = \left.\frac{\partial^2 f}{\partial y^2}\right|_{(0,0)}$$

Then

(A) $PS - QR > 0$ and $P < 0$

(B) $PS - QR > 0$ and $P > 0$

(C) $PS - QR < 0$ and $P > 0$

(D) $PS - QR < 0$ and $P < 0$

Q. 18 The area of the curved surface

$$S = \{(x,y,z) \in \mathbb{R}^3 : z^2 = (x-1)^2 + (y-2)^2\}$$

lying between the planes $z = 2$ and $z = 3$ is

(A) $4\pi\sqrt{2}$

(B) $5\pi\sqrt{2}$

(C) 9π

(D) $9\pi\sqrt{2}$

Q. 19 Let $a_n = \dfrac{1 + 2^{-2} + \cdots + n^{-2}}{n}$ for $n \in \mathbb{N}$. Then

(A) both the sequence (a_n) and the series $\sum_{n=1}^{\infty} a_n$ are convergent

(B) the sequence (a_n) is convergent but the series $\sum_{n=1}^{\infty} a_n$ is NOT convergent

(C) both the sequence (a_n) and the series $\sum_{n=1}^{\infty} a_n$ are NOT convergent

(D) the sequence (a_n) is NOT convergent but the series $\sum_{n=1}^{\infty} a_n$ is convergent

Q. 20 Let (a_n) be a sequence of real numbers such that the series $\sum_{n=0}^{\infty} a_n(x-2)^n$ converges at $x = -5$. Then this series also converges at

(A) $x = 9$

(B) $x = 12$

(C) $x = 5$

(D) $x = -6$

Q. 21 Let (a_n) and (b_n) be sequences of real numbers such that

$$|a_n - a_{n+1}| = \frac{1}{2^n} \quad \text{and} \quad |b_n - b_{n+1}| = \frac{1}{\sqrt{n}} \quad \text{for } n \in \mathbb{N}.$$

Then

(A) both (a_n) and (b_n) are Cauchy sequences

(B) (a_n) is a Cauchy sequence but (b_n) need NOT be a Cauchy sequence

(C) (a_n) need NOT be a Cauchy sequence but (b_n) is a Cauchy sequence

(D) both (a_n) and (b_n) need NOT be Cauchy sequences

Q. 22 Consider the family of curves $x^2 + y^2 = 2x + 4y + k$ with a real parameter $k > -5$. Then the orthogonal trajectory to this family of curves passing through $(2, 3)$ also passes through

(A) $(3, 4)$

(B) $(-1, 1)$

(C) $(1, 0)$

(D) $(3, 5)$

Q. 23 Consider the following statements:

I. Every infinite group has infinitely many subgroups.

II. There are only finitely many non-isomorphic groups of a given finite order.

Then

(A) both I and II are TRUE

(B) I is TRUE but II is FALSE

(C) I is FALSE but II is TRUE

(D) both I and II are FALSE

Q. 24 Suppose $f : (-1,1) \to \mathbb{R}$ is an infinitely differentiable function such that the series $\sum_{j=0}^{\infty} a_j \frac{x^j}{j!}$ converges to $f(x)$ for each $x \in (-1,1)$, where,

$$a_j = \int_0^{\pi/2} \theta^j \cos^j(\tan\theta)\,d\theta + \int_{\pi/2}^{\pi} (\theta-\pi)^j \cos^j(\tan\theta)\,d\theta$$

for $j \geq 0$. Then

(A) $f(x) = 0$ for all $x \in (-1,1)$

(B) f is a non-constant even function on $(-1,1)$

(C) f is a non-constant odd function on $(-1,1)$

(D) f is NEITHER an odd function NOR an even function on $(-1,1)$

Q. 25 Let $f(x) = \cos(x)$ and $g(x) = 1 - \frac{x^2}{2}$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then

(A) $f(x) \geq g(x)$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(B) $f(x) \leq g(x)$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(C) $f(x) - g(x)$ changes sign exactly once on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(D) $f(x) - g(x)$ changes sign more than once on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Q. 26 Let

$$f(x,y) = \iint_{(u-x)^2+(v-y)^2\leq 1} e^{-\sqrt{(u-x)^2+(v-y)^2}}\, du\, dv.$$

Then $\lim_{n\to\infty} f(n, n^2)$ is

(A) non-existent

(B) 0

(C) $\pi(1 - e^{-1})$

(D) $2\pi(1 - 2e^{-1})$

Q. 27 How many group homomorphisms are there from $\mathbb{Z}_2$ to S_5?

(A) 40

(B) 41

(C) 26

(D) 25

Q. 28 Let $y : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that y'' is continuous on $[0, 1]$ and $y(0) = y(1) = 0$. Suppose $y''(x) + x^2 < 0$ for all $x \in [0, 1]$. Then

(A) $y(x) > 0$ for all $x \in (0, 1)$

(B) $y(x) < 0$ for all $x \in (0, 1)$

(C) $y(x) = 0$ has exactly one solution in $(0, 1)$

(D) $y(x) = 0$ has more than one solution in $(0, 1)$

Q. 29 From the additive group $\mathbb{Q}$ to which one of the following groups does there exist a non-trivial group homomorphism?

(A) $\mathbb{R}^{\times}$, the multiplicative group of non-zero real numbers

(B) $\mathbb{Z}$, the additive group of integers

(C) $\mathbb{Z}_2$, the additive group of integers modulo 2

(D) $\mathbb{Q}^{\times}$, the multiplicative group of non-zero rational numbers

Q. 30 Let $f : \mathbb{R} \to \mathbb{R}$ be an infinitely differentiable function such that f'' has exactly two distinct zeroes. Then

(A) f' has at most 3 distinct zeroes

(B) f' has at least 1 zero

(C) f has at most 3 distinct zeroes

(D) f has at least 2 distinct zeroes

SECTION – B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q. 31 For each $t \in (0,1)$, the surface P_t in $\mathbb{R}^3$ is defined by

$$P_t = \{(x,y,z) : (x^2+y^2)z = 1,\ t^2 \le x^2+y^2 \le 1\}.$$

Let $a_t \in \mathbb{R}$ be the surface area of P_t. Then

(A) $a_t = \displaystyle\iint_{t^2 \le x^2+y^2 \le 1} \sqrt{1 + \frac{4x^2}{(x^2+y^2)^4} + \frac{4y^2}{(x^2+y^2)^4}}\, dx\, dy$

(B) $a_t = \displaystyle\iint_{t^2 \le x^2+y^2 \le 1} \sqrt{1 + \frac{4x^2}{(x^2+y^2)^2} + \frac{4y^2}{(x^2+y^2)^2}}\, dx\, dy$

(C) the limit $\lim\limits_{t \to 0^+} a_t$ does NOT exist

(D) the limit $\lim\limits_{t \to 0^+} a_t$ exists

Q. 32 Let $A \subseteq \mathbb{Z}$ with $0 \in A$. For $r, s \in \mathbb{Z}$, define

$$rA = \{ra : a \in A\}, \qquad rA + sA = \{ra + sb :\ a, b \in A\}.$$

Which of the following conditions imply that A is a subgroup of the additive group $\mathbb{Z}$?

(A) $-2A \subseteq A,\ \ A + A = A$

(B) $A = -A,\ \ A + 2A = A$

(C) $A = -A,\ \ A + A = A$

(D) $2A \subseteq A,\ \ A + A = A$

Q. 33 Let $y : (\sqrt{2/3}, \infty) \to \mathbb{R}$ be the solution of

$$(2x - y)y' + (2y - x) = 0,$$
$$y(1) = 3.$$

Then

(A) $y(3) = 1$

(B) $y(2) = 4 + \sqrt{10}$

(C) y' is bounded on $(\sqrt{2/3}, 1)$

(D) y' is bounded on $(1, \infty)$

Q. 34 Let $f : (-1, 1) \to \mathbb{R}$ be a differentiable function satisfying $f(0) = 0$. Suppose there exists an $M > 0$ such that $|f'(x)| \leq M|x|$ for all $x \in (-1, 1)$. Then

(A) f' is continuous at $x = 0$

(B) f' is differentiable at $x = 0$

(C) ff' is differentiable at $x = 0$

(D) $(f')^2$ is differentiable at $x = 0$

Q. 35 Which of the following functions is/are Riemann integrable on $[0,1]$?

(A) $f(x) = \int_0^x \left|\frac{1}{2} - t\right| dt$

(B) $f(x) = \begin{cases} x\sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

(C) $f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \cap [0,1] \\ -1 & \text{otherwise} \end{cases}$

(D) $f(x) = \begin{cases} x & \text{if } x \in [0,1) \\ 0 & \text{if } x = 1 \end{cases}$

Q. 36 A subset $S \subseteq \mathbb{R}^2$ is said to be *bounded* if there is an $M > 0$ such that $|x| \leq M$ and $|y| \leq M$ for all $(x,y) \in S$. Which of the following subsets of $\mathbb{R}^2$ is/are bounded?

(A) $\{(x,y) \in \mathbb{R}^2 : \ e^{x^2} + y^2 \leq 4\}$

(B) $\{(x,y) \in \mathbb{R}^2 : \ x^4 + y^2 \leq 4\}$

(C) $\{(x,y) \in \mathbb{R}^2 : \ |x| + |y| \leq 4\}$

(D) $\{(x,y) \in \mathbb{R}^2 : \ e^{x^3} + y^2 \leq 4\}$

Q. 37 Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as follows:

$$f(x,y) = \begin{cases} \dfrac{x^4 y^3}{x^6 + y^6} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Then

(A) $\lim_{t \to 0} \dfrac{f(t,t) - f(0,0)}{t}$ exists and equals $\dfrac{1}{2}$

(B) $\left.\dfrac{\partial f}{\partial x}\right|_{(0,0)}$ exists and equals 0

(C) $\left.\dfrac{\partial f}{\partial y}\right|_{(0,0)}$ exists and equals 0

(D) $\lim_{t \to 0} \dfrac{f(t,2t) - f(0,0)}{t}$ exists and equals $\dfrac{1}{3}$

Q. 38 Which of the following is/are true?

(A) Every linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ maps lines onto points or lines

(B) Every surjective linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ maps lines onto lines

(C) Every bijective linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ maps pairs of parallel lines to pairs of parallel lines

(D) Every bijective linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ maps pairs of perpendicular lines to pairs of perpendicular lines

Q. 39 Which of the following is/are linear transformations?

(A) $T:\mathbb{R}\to\mathbb{R}$ given by $T(x)=\sin(x)$

(B) $T: M_2(\mathbb{R})\to\mathbb{R}$ given by $T(A)=\text{trace}(A)$

(C) $T:\mathbb{R}^2\to\mathbb{R}$ given by $T(x,y)=x+y+1$

(D) $T: P_2(\mathbb{R})\to\mathbb{R}$ given by $T(p(x))=p(1)$

Q. 40 Let R_1 and R_2 be the radii of convergence of the power series $\sum_{n=1}^{\infty}(-1)^n x^{n-1}$ and $\sum_{n=1}^{\infty}(-1)^n \frac{x^{n+1}}{n(n+1)}$, respectively. Then

(A) $R_1 = R_2$

(B) $R_2 > 1$

(C) $\sum_{n=1}^{\infty}(-1)^n x^{n-1}$ converges for all $x\in[-1,1]$

(D) $\sum_{n=1}^{\infty}(-1)^n \frac{x^{n+1}}{n(n+1)}$ converges for all $x\in[-1,1]$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q. 41 Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined as follows:

$$f(x,y) = \begin{cases} (x^2-1)^2 \cos^2\left(\dfrac{y^2}{(x^2-1)^2}\right) & \text{if } x \neq \pm 1 \\ 0 & \text{if } x = \pm 1. \end{cases}$$

The number of points of discontinuity of $f(x,y)$ is equal to ____________.

Q. 42 Let $T : P_2(\mathbb{R}) \to P_4(\mathbb{R})$ be the linear transformation given by $T(p(x)) = p(x^2)$. Then the rank of T is equal to ____________.

Q. 43 If y is the solution of

$$y'' - 2y' + y = e^x,$$
$$y(0) = 0, \quad y'(0) = -1/2,$$

then $y(1)$ is equal to ____________. (rounded off to two decimal places)

Q. 44 The value of

$$\lim_{n\to\infty}\left(n\int_0^1 \frac{x^n}{x+1}\,dx\right)$$

is equal to ____________. (rounded off to two decimal places)

Q. 45 For $\sigma \in S_8$, let $o(\sigma)$ denote the order of σ. Then $\max\{o(\sigma) : \sigma \in S_8\}$ is equal to ____________.

Q. 46 For $g \in \mathbb{Z}$, let $\bar{g} \in \mathbb{Z}_8$ denote the residue class of g modulo 8. Consider the group $\mathbb{Z}_8^\times = \{\bar{x} \in \mathbb{Z}_8 : 1 \leq x \leq 7, \gcd(x,8) = 1\}$ with respect to multiplication modulo 8. The number of group isomorphisms from $\mathbb{Z}_8^\times$ onto itself is equal to ____________.

Q. 47 Let $f(x) = \sqrt[3]{x}$ for $x \in (0, \infty)$, and $\theta(h)$ be a function such that

$$f(3+h) - f(3) = hf'(3 + \theta(h)h)$$

for all $h \in (-1, 1)$. Then $\lim_{h \to 0} \theta(h)$ is equal to ____________.
(rounded off to two decimal places)

Q. 48 Let V be the volume of the region $S \subseteq \mathbb{R}^3$ defined by

$$S = \{(x, y, z) \in \mathbb{R}^3 : \ xy \le z \le 4, \ 0 \le x^2 + y^2 \le 1\}.$$

Then $\frac{V}{\pi}$ is equal to ____________. (rounded off to two decimal places)

Q. 49 The sum of the series $\sum_{n=1}^{\infty} \frac{2n+1}{(n^2+1)(n^2+2n+2)}$ is equal to ____________.
(rounded off to two decimal places)

Q. 50 The value of $\lim_{n \to \infty} \left(1 + \frac{1}{2^n} + \frac{1}{3^n} + \cdots + \frac{1}{(2023)^n}\right)^{\frac{1}{n}}$ is equal to ____________.
(rounded off to two decimal places)

Q. 51 – Q. 60 carry two marks each.

Q. 51 Let $f : \mathbb{R}^3 \to \mathbb{R}$ be defined as $f(x, y, z) = x^3 + y^3 + z^3$, and let $L : \mathbb{R}^3 \to \mathbb{R}$ be the linear map satisfying

$$\lim_{(x,y,z)\to(0,0,0)} \frac{f(1+x, 1+y, 1+z) - f(1,1,1) - L(x,y,z)}{\sqrt{x^2+y^2+z^2}} = 0.$$

Then $L(1, 2, 4)$ is equal to ____________. (rounded off to two decimal places)

Q. 52 The global minimum value of

$$f(x) = |x-1| + |x-2|^2$$

on $\mathbb{R}$ is equal to ____________. (rounded off to two decimal places)

Q. 53 Let $y : (1, \infty) \to \mathbb{R}$ be the solution of the differential equation

$$y'' - \frac{2y}{(1-x)^2} = 0$$

satisfying $y(2) = 1$ and $\lim_{x\to\infty} y(x) = 0$. Then $y(3)$ is equal to ____________.
(rounded off to two decimal places)

Q. 54 The number of permutations in S_4 that have exactly two cycles in their cycle decompositions is equal to ____________.

Q. 55 Let S be the triangular region whose vertices are $(0,0)$, $\left(0, \frac{\pi}{2}\right)$, and $\left(\frac{\pi}{2}, 0\right)$. The value of $\iint_S \sin(x)\cos(y)\, dx\, dy$ is equal to ____________.
(rounded off to two decimal places)

Q. 56 Let

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 3 \\ 1 & 1 & 4 & 4 & 4 \end{pmatrix}$$

and B be a 5×5 real matrix such that AB is the zero matrix. Then the maximum possible rank of B is equal to ____________.

Q. 57 Let W be the subspace of $M_3(\mathbb{R})$ consisting of all matrices with the property that the sum of the entries in each row is zero and the sum of the entries in each column is zero. Then the dimension of W is equal to ____________.

Q. 58 The maximum number of linearly independent eigenvectors of the matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

is equal to ____________.

Q. 59 Let S be the set of all real numbers α such that the solution y of the initial value problem

$$\frac{dy}{dx} = y(2-y),$$
$$y(0) = \alpha,$$

exists on $[0, \infty)$. Then the minimum of the set S is equal to ____________.
(rounded off to two decimal places)

Q. 60 Let $f : \mathbb{R} \to \mathbb{R}$ be a bijective function such that for all $x \in \mathbb{R}$, $f(x) = \sum_{n=1}^{\infty} a_n x^n$ and $f^{-1}(x) = \sum_{n=1}^{\infty} b_n x^n$, where f^{-1} is the inverse function of f. If $a_1 = 2$ and $a_2 = 4$, then b_1 is equal to ____________.